Quasistatic principles in the macroscopic electrodynamics of bianisotropic media

E. O. Kamenetskii
Faculty of Engineering, Department of Electrical Engineering—Physical Electronics, Tel Aviv University, Tel Aviv 69978, Israel
(Received 24 March 1998; revised manuscript received 9 September 1998)

Only very few theoretical results in macroscopic electrodynamics of bianisotropic composites have necessary experimental justifications. This fact, it seems, is not accidental. Artificial bianisotropic materials based on a composition of helices and Ω particles do not meet the requirements of macroscopic electrodynamics of condensed media. Our standpoint is based on the principle that in the description of the electromagnetic properties of a bianisotropic medium, one has to be able to separate the microscopic and macroscopic levels of consideration. In other words, specific properties of a bianisotropic medium should be defined separately from macroscopic Maxwell's equations. In this paper we formulate some principles that should underlie the main laws of macroscopic electrodynamics of bianisotropic media. Our consideration is based on the notion of two types of dual quasistatic (quasimagnetostatic and quasielectrostatic) bianisotropic particles.
[S1063-651X(98)10812-7]

PACS number(s): 41.20.−q, 03.50.De, 62.65.+k, 81.05.Zx

I. INTRODUCTION

At present, the electrodynamics of bianisotropic artificial media (and chiral media, as a particular case) is developing extensively. We are witnesses to a vast number of publications concerning different aspects of the electromagnetic theory of these media. For any newcomer in this area of science, two main theoretical questions, it seems, arise: (a) how to calculate the effective-medium (continuum) parameters (linear and nonlinear) of artificial bianisotropic media and (b) how to solve classical electrodynamics problems (scattering, diffraction, etc.) in different structures with bianisotropic materials. If, however, this newcomer is attentive enough, it will be seen that (at least in the microwave region) a large list of theoretical papers on the electrodynamics of bianisotropic media is accompanied by a not so high number of experimental works. Moreover, only few theoretical results have the necessary experimental justifications. Mainly, there are experiments to demonstrate the ability to rotate the plane of polarization of the electromagnetic wave, the reflection, and transmission coefficient measurements. One can see that these experiments are far-zone, or at least intermediate-zone, reflected field investigations. To the best of the author's knowledge, there are no experimental works on microwave bianisotropic structures (the structures that contain materials described by bianisotropic constitutive relations) to verify theoretical results concerning near-zone (quasistatic) reflected field investigations. Nobody has published experimental works of stratified bianisotropic media, dispersion characteristics of bianisotropic rectangular, or strip-line waveguide structures, field singularities at edges in bianisotropic media, etc. Is this lack of microwave experiments accidental? Is the fact that now, after ten years of intensive research in this area of science, no known microwave devices based on chiral and bianisotropic materials work as such accidental as well [1–5]?

In the microwave range, chiral and bianisotropic materials are particulate composites. Artificial chiral media (based on a composition of small helices in the host material) were developed to demonstrate the phenomenon of the electromagnetic activity at microwave frequencies, analogous to the optical activity. As a further generalization of such media, artificial bianisotropic materials based on a composition of helices and Ω particles were introduced as well. It becomes clear, however, that these composite materials are beyond the laws that form the basis of macroscopic electrodynamics of condensed media.

The main principles of macroscopic electrodynamics are based on the fact that one can separate the microscopic and macroscopic levels in the description of the electromagnetic properties of a medium. Because of the natural scale of lengths in media, one can use only those variables that have the Fourier-spectrum components in the k space (of the electromagnetic waves in a medium), up to some limiting cutoff wave number K ($1/K$ corresponds to the maximum scale of material nonhomogeneity). In other words, only those Fourier components with $k<K$ are relevant to the macroscopic problem. If

$$k \ll K, \tag{1}$$

we can almost certainly treat a system as a continuum. Relation (1) (which Robinson characterized as the truncation process [6]) is, as a matter of fact, the quasistatic approximation. The field vectors and constitutive parameters in the macroscopic Maxwell equations are obtained on the basis of the averaging procedure on scales much less than the macroscopic wavelength. Because of the averaging procedure, the electromagnetic boundary conditions can be introduced on interfaces of condensed media [7,8].

Do chiral materials and bianisotropic composites based on helices and Ω particles meet the requirements of the main principles of macroscopic electrodynamics? Are we able to separate the microscopic and macroscopic levels in the description of these materials, as it is usually assumed in macroscopic electrodynamics? One can see that in chiral materials and bianisotropic composites, the fact that the phase of the electromagnetic wave is different for different parts of a particle is essential, that is, nonlocal effects in these media are essential. A special feature of these materials is that we introduce constitutive relations that in fact connect the macroscopic field vectors *not separately from the macroscopic Maxwell equations*. For this reason, the parameters of chirality or bianisotropy are, at the same time, measures of nonlocal effects. The authors of numerous works where different electrodynamics problems with chiral and bianisotropic materials (described by the constitutive relations) are solved formally analyze these problems as if no aspects of nonlocality exist. Here are some curious examples.

There is a well known problem in electrodynamics concerning correct boundary conditions to match fields on two sides of the layer with nonlocal properties. One has to take into account that because of the effects of spatial dispersion the order of differential equations increases. This demands the use of the so-called additional boundary conditions (together with the electrodynamical boundary conditions). Such boundary conditions do not have a universal character and are found on the basis of the microscopic theory. For different types of interfaces (in plasma, ferromagnetics, and optically active crystals), one has different additional boundary conditions [8–11]. For chiral and Ω materials, this problem is solved very ''simply.'' One can see, for example, that the symmetrical form of the Drude-Born-Fedorov constitutive relations makes it possible to obtain the boundary conditions requiring the continuity of tangential components of the **E** and **H** fields across the bimaterial interface, but not continuity of the normal components of the **D** and **B** fields. This is explained by Lakhtakia (see [2], p. 136) as follows. The boundary conditions on the normal components of the **D** and **B** fields are necessary and sufficient for static problems. Because electromagnetic chirality cannot exist for static problems, it is very satisfying that the Maxwell curl postulates, unaided by extraneous considerations, give rise to the necessary and sufficient boundary conditions that involve only the **E** and **H** fields.

This statement infringes the laws of macroscopic electrodynamics, which require four boundary conditions to be satisfied: two for the tangential components of the **E** and **H** fields and two for the normal components of the **D** and **B** fields. If the boundary conditions involve only the **E** and **H** fields, one cannot distinguish, for example, the problems with and without a surface charge density at the interface. Moreover, what does the continuity equation for charge density and current density mean in such a case? It becomes apparent that many boundary problems solved recently for chiral media and bianisotropic composites (for example, an analysis of singularities in Green's dyadics [12–15], field singularities at edges [16], [17], waveguide step discontinuities [18], and many other boundary problems) are far from physical reality and may be considered as examples of interesting but abstract (from a physical point of view) approaches. Doubt is cast on the validity of the formal extension of the known electrodynamical problems solved for isotropic and anisotropic materials with local properties to such nonlocal media as chiral media and bianisotropic composites based on helices and Ω particles.

We can see that because of the space-resonance properties of composites based on helices and Ω particles (caused by the first-order role played by the size parameters qa in the emergence of the magnetoelectric properties; here a is the particle size and q is the wave number in the host material), it is impossible to use the procedure of wave-number truncation and the averaging procedure. The quasistatic approximation (1) does not work in this case. An analysis of the known properties of chiral and Ω particles shows that these particles are not quasistatic oscillators. Only in the far-zone region can one consider these particles as a combination of the electric and magnetic dipoles. In the near-field zone, we do not have a combination of such dipoles [19–21]. This is the main reason why constitutive relations used for composites based on helices or Ω particles characterize, in fact, the gas of scatterers. We have an example of a phenomenological description of a diffraction structure rather than a continuous medium. The laws of macroscopic electrodynamics are not completely applicable in this case. This puts forth questions about generalized susceptibilities, energy relations, boundary conditions, etc., for microwave bianisotropic media.

It becomes clear that for bianisotropic artificial media, the main propositions that underlie the laws of macroscopic electrodynamics should be considered. In contrast to dilute bianisotropic composites based on helices and Ω particles, we introduce the notion of *condensed matter* bianisotropic composites. Evidently, one cannot talk at present about the macroscopic electrodynamics of microwave bianisotropic media until the electrodynamically macroscopic (condensed matter) bianisotropic materials have been synthesized. This problem, however, cannot be solved until the main theoretical principles of condensed matter bianisotropic composite materials have been formulated. Our standpoint is based on the principles that specific properties of a bianisotropic medium have to be defined separately from macroscopic Maxwell's equations. Because of this possibility to separate the field and medium equations (in other words, to separate the microscopic and macroscopic levels in the description of the electromagnetic properties of a medium), the so-called effects of temporal and spatial dispersion can be considered for media characterized by specific time and space scales. The main point is that the constitutive parameters of bianisotropic media should be described *quasistatically* (quasimagnetostatically or quasielectrostatically).

The main concept that underlies our microscopic description of bianisotropic media is *the concept of magnetostatically and electrostatically controlled bianisotropic particles*. These particles are small resonance structures with short-wavelength (for example, magnetostatic waves in ferromagnetics or elastodynamic quasielectrostatic waves in piezoelectrics) oscillations. While the magnetostatically controlled bianisotropic materials (MCBMs) have been considered in previous works [22–24], an analysis of the electrostatically controlled bianisotropic materials (ECBMs) is a different proposition by the present author [25].

The paper is organized as follows. In Sec. II we show that the so-called integral-form constitutive relations (ICRs) for bianistropic media considered in many publications can be used only when microscopically defined generalized susceptibilities are applicable. As we will show, it is possible for such ''local media'' as the MCBMs or ECBMs, but impossible for bianistropic media with nonlocal properties (based on helices or Ω particles). The electromagnetic field energy in microwave bianisotropic media is the subject of Sec. III. In many works, the energy relations in microwave bianisotropic media are considered as just an extension of similar relations used for anisotropic media. A careful analysis, however, shows that the situation with bianisotropic media is fundamentally different. The notion of quasistatically controlled bianisotropic media is essential in this consideration. Section IV is devoted to a discussion about the dynamical perturbation of a system of quasistatic bianisotropic particles, which is a composition of microscopic oscillators. Section V contains concluding remarks.

II. INTEGRAL-FORM CONSTITUTIVE RELATIONS AND GENERALIZED SUSCEPTIBILITIES IN BIANISOTROPIC MEDIA

It is known that the effect of optical activity may be characterized in two ways: as a separate phenomenon in so-called chiral media (see [1,2] and numerous references in these books) or as a particular case of a general effect of spatial dispersion in dielectric media [8,9]. For temporally and spatially dispersive dielectric media, the ICRs are used. When this medium is time invariant and spatially homogeneous, the dielectric tensor $\boldsymbol{\epsilon}(\omega,\mathbf{k})$ can be introduced. In the long-wavelength approximation, the temporally and spatially dispersive dielectric medium becomes only temporally dispersive, that is, the quasistatic limit ($|\mathbf{k}|\to 0$) exists. One can take advantage of the power-series expansion of the constitutive tensor over $\mathbf{k}$ in the region near $|\mathbf{k}|=0$ [8,9]. We also have other examples. In disorder dielectric composites, when the effects of spatial dispersion are taken into account, the long-wavelength (quasistatic) limit is used to compute the effective permittivity tensor [26–28].

A calculation of tensor $\boldsymbol{\epsilon}(\omega,\mathbf{k})$ in continuous media is based on a microscopic theory. It may be the quantum mechanical theory based on a perturbation theory and the notion of a generalized susceptibility [8,11,29]. In dipole crystal lattices, Edward's method of separation of the long-wavelength (macroscopic) and the short-wavelength (microscopic) electric fields may also be applied [30]. In the present author's recent work [31], the so-called sampling theorem was used to develop a dynamical theory and calculate the effective parameters of dielectric crystal lattices.

The ICRs for bianisotropic media may be formally introduced in various forms for example, as

$$D_i(t,\mathbf{r})=(\alpha_{ij}\circ E_j)+(\beta_{ij}\circ B_j),$$
$$H_i(t,\mathbf{r})=(\gamma_{ij}\circ E_j)+(\nu_{ij}\circ B_j) \tag{2}$$

or

$$D_i(t,\mathbf{r})=(\epsilon_{ij}\circ E_j)+(\xi_{ij}\circ H_j),$$
$$B_i(t,\mathbf{r})=(\zeta_{ij}\circ E_j)+(\mu_{ij}\circ H_j). \tag{3}$$

On the right-hand sides of expressions (2) and (3), we have the integral operators similar to the integral operator

$$\epsilon_{ij}\circ E_j=\int_{-\infty}^{t}dt'\int d\mathbf{r}'\,\epsilon_{ij}(t,\mathbf{r},t',\mathbf{r}')E_j(t',\mathbf{r}'). \tag{4}$$

When a bianisotropic medium is time invariant and spatially homogeneous, the ICRs have a temporal and space convolution form. For example, for Eq. (3) one can write

$$\mathbf{D}(\omega,\mathbf{k})=\boldsymbol{\epsilon}(\omega,\mathbf{k})\cdot\mathbf{E}(\omega,\mathbf{k})+\boldsymbol{\xi}(\omega,\mathbf{k})\cdot\mathbf{H}(\omega,\mathbf{k}),$$
$$\mathbf{B}(\omega,\mathbf{k})=\boldsymbol{\zeta}(\omega,\mathbf{k})\cdot\mathbf{E}(\omega,\mathbf{k})+\boldsymbol{\mu}(\omega,\mathbf{k})\cdot\mathbf{H}(\omega,\mathbf{k}). \tag{5}$$

To the best of the author's knowledge the ICRs for bianisotropic media were introduced for the first time by Hornreich and Shtrikman for a hypothetical situation of the optical effect of gyrotropy in magnetoelectric crystals [32]. Nobody has realized this situation in physical experiments. The present author used the ICRs to analyze the energy balance equation in bianisotropic media [33]. Lakhtakia and Weiglhofer formally considered the ICRs in bianisotropic media as "the most general linear relations that can describe any linear medium—indeed, the entire universe after linearization" [34]. In [35], the ICRs were used in the theory of perturbation for an analysis of the effective constitutive parameters in a bi-isotropic continuum.

Our standpoint is that we should try to clarify the physical essence of the phenomena and only afterward build a mathematical approach based on the physical theory. Otherwise, one may obtain abstract mathematical exercises that are far from physical reality. First of all, the question arises, what are the physical assumptions to have the convergence of integrals in the ICRs for time-invariant and spatially homogeneous bianisotropic media? The reaction of a causal medium is dependent on previous values of the fields because of the finiteness of a time reorganization of all the system of dipoles. In fact, such a "memory" is retained during the time of system relaxation T_r. Therefore, the response functions decrease rapidly for $t-t'\gg T_r$. On the other hand, we may have a nonlocal connection between the fields and the system reaction. The response functions have to decrease when the difference $|\mathbf{r}-\mathbf{r}'|$ increases. All this means that we have to consider the influence of *short-time and short-space electrostatic and magnetostatic interactions* between particles on the polarization properties of a bianisotropic medium. In this case, the kernels $\epsilon_{ij}(t',\mathbf{r}')$, $\xi_{ij}(t',\mathbf{r}')$, $\zeta_{ij}(t',\mathbf{r}')$, and $\mu_{ij}(t',\mathbf{r}')$ of the integral operators in Eq. (3) (and, similarly, the kernels of the operators in other forms of the ICRs) are the "responses" of a medium to the δ-function electric and magnetic fields.

Based on our consideration, we introduce the notion of the *local temporally dispersive bianisotropic media* (LTDBM). The LTDBM are characterized by constitutive parameters satisfying the long-wavelength (quasistatic) limit. This means that for constitutive tensors used, for example, in Eq. (5), one can write [36]

$$\boldsymbol{\epsilon}(\omega,\mathbf{k})|_{|\mathbf{k}|\to 0}\to\boldsymbol{\epsilon}(\omega),\quad \boldsymbol{\xi}(\omega,\mathbf{k})|_{|\mathbf{k}|\to 0}\to\boldsymbol{\xi}(\omega),$$
$$\boldsymbol{\zeta}(\omega,\mathbf{k})|_{|\mathbf{k}|\to 0}\to\boldsymbol{\zeta}(\omega),\quad \boldsymbol{\mu}(\omega,\mathbf{k})|_{|\mathbf{k}|\to 0}\to\boldsymbol{\mu}(\omega). \tag{6}$$

When the limit (6) takes place, we can take advantage of the power-series expansion of the constitutive tensors over $\mathbf{k}$ in the region near $|\mathbf{k}|=0$. That is, the effects of spatial dispersion are considered as *small-order effects*, similar to the approach used in [8–11] for isotropic and anisotropic media.

A full answer to the question whether the LTDBM can be physically realized or not should be found from a microscopic analysis. To consider the problem of the ICRs on the microscopic level, let us take advantage of the notion of a generalized susceptibility widely used in macroscopic electrodynamics. In the case of dynamical perturbations of a system, an external action can be described by the exciting operator in the total Hamiltonian [8,11,29]

$$\hat{\mathcal{H}}'(t) = -\hat{x}_i f_i(t), \tag{7}$$

where x_i is a series of quantities characterizing a system, $\hat{x}$ is an operator of a given physical quantity, and $f_i(t)$ are generalized forces dependent on time. The average quantities $\overline{x(t)}$ are represented by a linear integral-operator form

$$\overline{x_i}(t) = \int_{-\infty}^{t} dt' \alpha_{ij}(t-t') f_j(t'). \tag{8}$$

One can rewrite this relation for the Fourier components

$$\overline{x_i}(\omega) = \alpha_{ij}(\omega) f_j(\omega). \tag{9}$$

The function α_{ij} completely defines the behavior of a system under an external action. This function is called generalized susceptibility. When a linear integral relation (8) has the necessary physical foundation, the time variation of the energy (caused by the external excitation) may be expressed as

$$\frac{dW}{dt} = -\overline{x_i} \frac{df_i}{dt}. \tag{10}$$

For a dielectric medium, the function f may be interpreted as the external electric field and x as the electric dipole moment induced in this field [8]. Now we extend our consideration to the general case of a bianisotropic medium. When the fields of interactions between particles are described by scalar potentials, one can write for the exciting operator in the total Hamiltonian

$$\hat{\mathcal{H}}'(t) = -E_i(t)\hat{p}_i - H_j(t)\hat{m}_j, \tag{11}$$

where $\hat{p}$ and $\hat{m}$ are operators of electric and magnetic moments, respectively. The average quantities are represented as

$$\overline{p_i}(t) = \int_{-\infty}^{t} dt' a_{ij}(t-t') E_j(t') + \int_{-\infty}^{t} dt' b_{ij}(t-t') H_j(t),$$
$$\overline{m_j}(t) = \int_{-\infty}^{t} dt' c_{jk}(t-t') E_k(t') + \int_{-\infty}^{t} dt' d_{jk}(t-t') H_k(t'). \tag{12}$$

Based on relations (12), one can write, as a result, integral-form relations (for temporally dispersive bianisotropic media). So the ICRs formally defined for bianisotropic media can be reduced to microscopically defined generalized susceptibilities. This is possible when a bianisotropic particle is found in energy eigenstates and the particle fields can be described by the Hamiltonian and the exciting operator in the total Hamiltonian of a system of particles can be introduced.

The notion of generalized susceptibilities obtains a peculiar sense for the electromagnetic field energy in media. The time variation of the electromagnetic energy of a bianisotropic body is expressed as

$$\int_V \left(\mathbf{E} \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H} \frac{\partial \mathbf{B}}{\partial t} \right) dV.$$

We correlate this expression with the expression (10) for the time variation of the energy caused by the external excitation. One can consider the components of vectors **D** and **B** as two generalized forces and the components of **E** and **H** as two series of quantities characterizing a system. Let us suppose that constitutive relations are written as

$$E_i(t) = \int_{-\infty}^{t} dt' e_{ij}(t-t') D_j(t') + \int_{-\infty}^{t} dt' f_{ij}(t-t') B_j(t'),$$
$$H_i(t) = \int_{-\infty}^{t} dt' g_{ij}(t-t') D_j(t') + \int_{-\infty}^{t} dt' h_{ij}(t-t') B_j(t'). \tag{13}$$

Similarly to an analysis made in [8] for anisotropic media, one can obtain the symmetry relations of constitutive tensors in bianisotropic media. When the components of the **E** field from one side and the components of the **H** field from the other side are considered independent, we can use the principle of kinetic coefficient symmetry (the Onsager-Casimir principle) [37,38] to characterize the symmetry properties for constitutive tensors. In this case, we have for tensors in expression (13)

$$\mathbf{e}_{ij}(\omega, \mathbf{H}_0) = \mathbf{e}_{ji}(\omega, -\mathbf{H}_0),$$
$$h_{ij}(\omega, \mathbf{H}_0) = h_{ji}(\omega, -\mathbf{H}_0), \tag{14}$$
$$f_{ij}(\omega, \mathbf{H}_0) = -g_{ji}(\omega, -\mathbf{H}_0),$$

where $\mathbf{H}_0$ is the external bias magnetic field. For spatially dispersive bianisotropic media [33], these relations can be extended as

$$e_{ij}(\omega, \mathbf{k}, \mathbf{H}_0) = e_{ji}(\omega, -\mathbf{k}, -\mathbf{H}_0),$$
$$h_{ij}(\omega, \mathbf{k}, \mathbf{H}_0) = h_{ji}(\omega, -\mathbf{k}, -\mathbf{H}_0), \tag{15}$$
$$f_{ij}(\omega, \mathbf{k}, \mathbf{H}_0) = -g_{ji}(\omega, -\mathbf{k}, -\mathbf{H}_0).$$

So for constitutive relations written as

$$\mathbf{E}(\omega, \mathbf{k}) = \mathbf{e}(\omega, \mathbf{k}) \cdot \mathbf{D}(\omega, \mathbf{k}) + \mathbf{f}(\omega, \mathbf{k}) \cdot \mathbf{B}(\omega, \mathbf{k}),$$
$$\mathbf{H}(\omega, \mathbf{k}) = \mathbf{g}(\omega, \mathbf{k}) \cdot \mathbf{D}(\omega, \mathbf{k}) + \mathbf{h}(\omega, \mathbf{k}) \cdot \mathbf{B}(\omega, \mathbf{k}), \tag{16}$$

the Onsager-Casimir principle gives

$$\tilde{\mathbf{e}} = \mathbf{e}^T,$$
$$\tilde{\mathbf{h}} = \mathbf{h}^T,$$
$$\tilde{\mathbf{f}} = -\mathbf{g}^T. \tag{17}$$

Here a tilde denotes the time reversal and T denotes transposition.

One obtains the following relationship between constitutive tensors in expressions (5) and (16):

$$[\mathbf{I} - \mathbf{e} \cdot \boldsymbol{\epsilon} - \mathbf{f} \cdot \boldsymbol{\zeta}]^{-1} \cdot [\mathbf{e} \cdot \boldsymbol{\xi} + \mathbf{f} \cdot \boldsymbol{\mu}]$$
$$- [\mathbf{g} \cdot \boldsymbol{\epsilon} + \mathbf{h} \cdot \boldsymbol{\zeta}]^{-1} \cdot [\mathbf{I} - \mathbf{g} \cdot \boldsymbol{\xi} - \mathbf{h} \cdot \boldsymbol{\mu}] = 0,$$

where **I** is the unit dyadic. It is not entirely evident that symmetry relations for tensors in expression (5) will have a form similar to relations (16). So the formal use of the Onsager-Casimir principle for constitutive tensors in expression (5) (as it has been done in [39,40]) can hardly be admissible. We have shown ''macroscopically'' that the ICRs are applicable only for LTDBM. Based on the notion of generalized susceptibilities, we also have shown how the ICRs are physically admissible on a microscopic level. Now let us apply our analysis to the known types of bianisotropic media.

We can see that LTDBM are distinguished, in principle, from natural magnetoelectric crystals [41,42] and piezoelectric-piezomagnetic magnetoelectric composites [43] (which are, as a matter of fact, nontemporally dispersive materials), chiral media and bianisotropic composites based on helices and Ω particles [1,2] (which are nonlocal media), and moving dielectric-magnetic materials [41,42]. So the ICRs cannot be applicable for these media.

One can also find other arguments against the possibility of using the ICRs in nonlocal bianisotropic media. In comments [45] on Ref. [44] it was pointed out that in nonlocal bianisotropic media, the kernels in cross terms in the ICRs are not independent, but proportional to that in the coterms and therefore this leads to double inclusion of the same effects in the model. Formal use of the ICRs without necessary physical justification may lead to incorrect results. For example, doubt is cast on the validity of the effective constitutive operators used in [35] to calculate the long-wavelength effective parameters of chiral media.

There are other arguments that arise from the notion of a generalized susceptibility. Bi-isotropic media, natural magnetoelectric crystals, and known microwave chiral and bianisotropic composite materials based on helices and Ω particles do not meet one of the main requirements of macroscopic electrodynamics of causal condensed media: separation of the microscopically and macroscopically dynamical levels of consideration. For these media, one cannot describe a dynamical perturbation of a microscopic system by the exciting operator in the total Hamiltonian and therefore introduce the notion of a generalized susceptibility. So the ICRs are not physically admissible for such media.

We have another situation for MCBMs and ECBMs. The MCBM particle and the ECBM particle are the *quasistatic oscillators that are found in energy eigenstates*. One can distinguish the internal fields of a particle and the external fields and consider excitation of quasistatic oscillations by the external fields [22–25]. Since the MCBMs and ECBMs are characterized by the quasistatic processes, these media are LTDDM. One can obtain the Hamiltonian of every particle and describe the dynamical perturbation of a system by the exciting operator in the total Hamiltonian. So the ICRs and the notion of generalized susceptibilities are physically admissible for the MCBMs and the ECBMs. The Onsager-Casimir principle (17) is applicable to this case.

III. THE ELECTROMAGNETIC FIELD ENERGY IN BIANISOTROPIC MEDIA

An analysis of the energy balance equation is one of the powerful tools to investigate electromagnetic wave propagation in media. For bianisotropic media, the notion of LTDBM plays a very important role in a physical meaning of the energetic relations. Several important microscopic aspects of LTDBM become understandable based on the energetic relations.

When a medium is characterized by constitutive parameters and the wave process is described by Maxwell's equations, the well known procedure leads us to the energy balance equation. For anisotropic media with local properties, one has [8]

$$-\vec{\nabla}\cdot\overline{\vec{S}}=\frac{\partial\overline{W}}{\partial t}+\overline{\mathcal{P}}. \tag{18}$$

There are fundamental issues that one should be aware of. One can interpret the terms in Eq. (18) only for weakly absorbing media. In this case $\overline{\vec{S}}$ is interpreted as the average (on the period $2\pi/\omega$) Poynting vector, $\overline{W}$ as the average density of the energy, and $\overline{\mathcal{P}}$ as the average density of the dissipation losses. The dissipation itself may not be so weak, but this dissipation has to be strongly reduced by choosing the corresponding frequency region. This means that the field must be closed to a monochromatic field. On the other hand, to derive the energy balance equation, it is insufficient to consider pure monochromatic fields since no accumulation of the electromagnetic energy takes place in this case. Therefore, the quasimonochromatic electromagnetic fields are usually considered. The average density of the energy in Eq. (18) has a physical meaning of the internal energy of a body in the electromagnetic field similarly to the energy of a body in the constant electric and magnetic fields [8].

Energetic relations for chiral and bianisotropic media have been the subject of many publications. In [1,2,42] the energy balance equations were considered for the time-harmonic fields; in [46,47] we have investigations for the quasimonochromatic fields in lossless temporally dispersive bianisotropic media. The analyses in [46,47] are based on the assumption that one can obtain quadratic forms in Poynting's theorem for bianisotropic media similarly to the terms in Eq. (18) obtained for anisotropic media. Obviously, the result of [46,47] may be interpreted as just an extension of a similar expression for the energy for bianisotropic media.

The situation, however, is not so trivial. A careful analysis of averaged quadratic forms in Poynting's theorem [33] shows that the energy balance in bianisotropic media is determined not only by the material constitutive parameters but also by the structure of the electromagnetic field. Taking into account that a tensor of the second rank A_{ij} can be written as a sum of Hermitian A^{H}_{ij} and anti-Hermitian A^{aH}_{ij} tensors, one can see that for any two tensors A_{ij} and B_{ij}, a contraction $A^{H}_{ij}B^{aH}_{ij}$ is an imaginary quantity and, at the same time, contractions $A^{H}_{ij}B^{H}_{ij}$ and $A^{aH}_{ij}B^{aH}_{ij}$ are real quantities. This known fact leads us [for the quasimonochromatic fields $\mathbf{E}=\mathbf{E}_m(t)e^{i\omega t}$ and $\mathbf{H}=\mathbf{H}_m(t)e^{i\omega t}$] to the following representation of Poynting's theorem for temporally dispersive bianisotropic media with constitutive tensors $\epsilon(\omega)$, $\xi(\omega)$, $\zeta(\omega)$, and $\mu(\omega)$ [33]:

$$-\vec{\nabla}\cdot\overline{\vec{S}}=\overline{Q}(\omega,t)+\overline{\mathcal{P}}. \tag{19}$$

Equation (19) does not have the continuity-equation form of Eq. (18). To reduce Eq. (19) to the form of Eq. (18), necessary constraints to the structure of the quasimonochromatic field have to be imposed. These constraints are [33]

$$E^*_{m_i}\frac{\partial H_{m_j}}{\partial t}=H_{m_j}\frac{\partial E^*_{m_i}}{\partial t}. \tag{20}$$

Based on Eq. (20), one can represent the term Q in Eq. (19) as

$$\overline{Q}(\omega,t)=\frac{\partial\overline{W}}{\partial t}, \tag{21}$$

where

$$\overline{W}=\frac{1}{4}\left\{\frac{\partial(\omega\epsilon^H_{ij})}{\partial\omega}E^*_iE_j+\frac{\partial(\omega\mu^H_{ij})}{\partial\omega}H^*_iH_j+\frac{\partial[\omega(\zeta^H_{ij}+\xi^H_{ij})]}{\partial\omega}(H^*_iE_j)^H+\frac{\partial[\omega(\zeta^{aH}_{ij}-\xi^{aH}_{ij})]}{\partial\omega}(H^*_iE_j)^{aH}\right\}. \tag{22}$$

Thus, when constraints (20) take place we can rewrite Eq. (19) in the form of Eq. (18). In this case, we may identify $\overline{\mathcal{P}}$ as a term that describes dissipative losses. The lossless case may be characterized by different systems of relations. For example, the first system is

$$\boldsymbol{\epsilon}=\boldsymbol{\epsilon}^+,\quad \boldsymbol{\mu}=\boldsymbol{\mu}^+,\quad \boldsymbol{\xi}=\boldsymbol{\zeta}^+, \tag{23}$$

where the plus superscript denotes the transpose and the complex conjugate procedure. The second system may be written, for example, as

$$\boldsymbol{\epsilon}=\boldsymbol{\epsilon}^+,\quad \boldsymbol{\mu}=\boldsymbol{\mu}^+,\quad (\zeta^H_{ij}-\xi^H_{ij})(H^*_iE_j)^{aH}+(\zeta^{aH}_{ij}+\xi^{aH}_{ij})(H^*_iE_j)^H=0. \tag{24}$$

Relations (23) are well known [1,42,46,47]. These relations mean that the term $\overline{\mathcal{P}}$ vanishes for all possible **E** and **H** fields. In contrast, relations (24) demonstrate a certain correlation between the field structure and the constitutive parameters of a medium that provides nondissipative propagation of the electromagnetic wave. For lossless or weakly absorbing media, one can conventionally interpret the term $\overline{W}$ described by Eq. (22) as the average density of the energy. The main feature is that this term is defined not only by the structure of a medium (the structure of the constitutive parameters) but also by the structure of the electromagnetic field.

A quadratic form makes sense of the energy when the process of variation of this quadratic form has a certain physical interpretation. In our case, this physical interpretation is contained in Poynting's theorem (18). In anisotropic media, variation of the energy in a given point is due to variation of amplitudes of the electric and/or magnetic fields and these variations could be completely independent of each other. To vary the energy term $\overline{W}$ in bianisotropic media, a certain time-domain correlation between amplitudes of the electric and the magnetic fields is essential. Necessary constraints on the structure of the amplitudes of the quasimonochromatic field could be imposed since these amplitudes are slow (in comparison to the period $2\pi/\omega$) functions of time.

There is an evident possibility to represent constraints (20) as

$$\frac{E^*_{m_i}(t)}{H_{m_j}(t)}=\frac{\partial E^*_{m_i}(t)/\partial t}{\partial H_{m_j}(t)/\partial t}=\frac{dE^*_{m_i}(t)}{dH_{m_j}(t)}, \tag{25}$$

where $dE^*_{m_i}(t)$ and $dH_{m_j}(t)$ are differentials of $E^*_{m_i}(t)$ and $H_{m_j}(t)$, respectively. Expression (25) may be written in a linear form

$$E^*_{m_i}(t)=F_{ij}H_{m_j}(t). \tag{26}$$

The tensor F_{ij}, being time independent, characterizes the structure of the quasimonochromatic field.

Let us consider a particular case when

$$\mathbf{E}_m(t)=\mathbf{E}_0e^{-i\omega_1 t},\quad \mathbf{H}_m(t)=H_0e^{i\omega_1 t}, \tag{27}$$

where $\mathbf{E}_0$ and $\mathbf{H}_0$ are constant vectors and $\omega_1\ll\omega$. It is evident that in this case the constraints (20) and (26) are satisfied identically.

The physical meaning of the constraints (20) and (26) is quite clear. Let for time t_1 a system have the density of the energy $\overline{W}^{(1)}$, which is defined by expression (22). For certain constitutive parameters, this energy may be obtained from different structures (configurations) of the quasimonochromatic field. We conventionally denote these field structures as states $a,b,\ldots$. Let for time t_2 the system have the density of the energy $\overline{W}^{(2)}$. In accordance with the field constraints, the transition $\overline{W}^{(1)}\rightarrow\overline{W}^{(2)}$ should be accompanied not only by variation of amplitudes of the fields but by rebuilding of the field structures as well. New field structures (we suppose that constitutive parameters are invariable) should be characterized as states $a',b',\ldots$, which are obtained due to certain rules (field constraints) for rebuilding the field structures: $a\rightarrow a', b\rightarrow b'\ldots$. Our explanations may be illustrated by the qualitative diagram ($\mathrm{Re}\,E^*_{m_i}$ versus $\mathrm{Re}\,H_{m_j}$), shown in Fig. 1. In this diagram the transitions $a\rightarrow a'$ and $b\rightarrow b'$ are depicted by straight lines [because of relations (26)]. The lines $\overline{W}^{(1)}$ and $\overline{W}^{(2)}$ correspond to different levels of the energy.

Contrary to anisotropic media (see [8], where thermodynamic aspects of the internal energy of a body in the high-frequency electromagnetic field are discussed), the average density of the energy $\overline{W}$ for microwave bianisotropic media does not have the physical meaning of the density of the internal energy in the electromagnetic field, similar to the density of the internal energy in constant electric and mag-

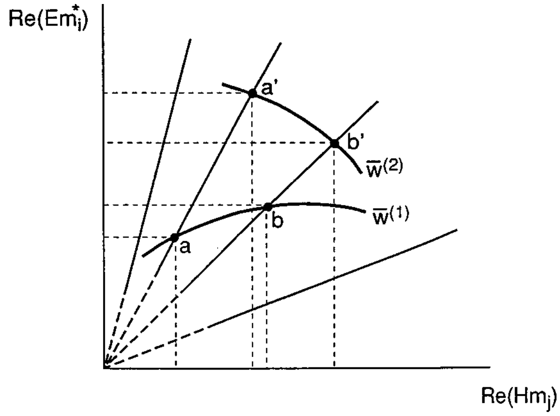

FIG. 1. Qualitative diagram of the field structure rebuildings correlated with the energy variation.

netic fields. Dzyaloshinskii has found the free energy of a magnetoelectric medium in the constant electric and magnetic fields [48]. It is not completely evident that, for microwave bianisotropic media, the average density $\overline{W}$ expressed by Eq. (22) can be physically reduced to the free energy described by Dzyaloshinskii when $\omega\rightarrow 0$. That is why it is impossible to introduce *a priori* the notion of the density of the internal energy of microwave bianisotropic materials in the electromagnetic field and to derive conditions for constitutive parameters based on the positive definite energy function (as it has been done in [46,47]). Another argument that the average energy density for causal bianisotropic materials cannot be reduced to Dzyaloshinskii's free energy has been adduced in [49,50].

One can correlate the properties of nonthermodynamic fluctuations with the quantities characterizing the behavior of a body under certain actions of time-dependent external fields. This is possible when we are able to distinguish a part of the total Hamiltonian correlated with external forces [8,29]. In bianisotropic composites described in [22–24], every small particle is a ferromagnetic resonator with short-wavelength magnetostatic wave oscillations. These magnetostatically controlled oscillating processes in a bianisotropic particle have scales of space variations much less (about two to four order of magnitudes) than the corresponding scales of the external electric and magnetic fields. This makes it possible to distinguish the intrinsic magnetostatic mode fields and the external fields [10,51]. That is why, in these composite materials (which are in fact *oscillatory media*), we can obtain the Hamiltonian of every particle of a system and distinguish a part of the total Hamiltonian correlated with the external fields. It becomes clear that for such bianisotropic composites as MCBMs [22–24], the term $\overline{W}$ in Eq. (18) has the physical meaning of the average density of electromagnetic energy. The same consideration is applicable for the ECBMs [25].

IV. DISCUSSION

The MCBMs and the ECMBs are compositions of microscopic oscillators and the quantum mechanical models can be applicable to describe dynamical perturbations of a system of these oscillators by an external action. The apparatus of a density matrix, well developed in quantum mechanics, may be used to find averaged dyadic polarizabilities of the MCBMs or ECBMs. It demands the solution of the Liouville equation for the density matrix of a bianisotropic body excited by the external electric and magnetic fields similar to an analysis made for a dielectric body excited by the external electric field [52–54]. To make such an analysis, a quantum theory of the interaction of quasistatic bianisotropic media with the external fields has to be developed. This general theory should be a subject for future efforts. We will touch upon only some of the main aspects of this treatment.

One can suppose that to define energy levels of every bianisotropic particle, a symmetry analysis should be used similar to an analysis applied for the classification of molecular terms [52]. This analysis is impossible, however, for bianisotropic particles that exist in two different configurations that cannot be superimposed by the application of rotations or translations (enantiomers). In a quantum mechanics description of optical activity, the remaining questions are, Can the existence of optically active (chiral) molecules be explained in quantum mechanics? Why are chiral molecules never found in energy eigenstates? There are many efforts to account for the instability of chiral molecules [55,56]. Thus, in our case, one should be aware that the classification of energy eigenstates based on a symmetry analysis is applicable only for bianisotropic particles without symmetry breaking. However, can an analysis of bianisotropic particles without symmetry breaking be carried out similarly to an analysis applied for classification of molecular terms? Let us turn our attention to a bianisotropic particle based on geometrically symmetrical magnetostatic wave resonators with surface metallization described in [22,23]. This particle can be represented as a glued pair of magnetic and electric dipoles (the magnetic dipole is due to the body of a ferromagnetic resonator and the electric dipole is due to special-form surface metallization). Thus an apparent question arises, Can one consider this particle as a combination of two (initially independent) resonance structures? The first structure is a straight-edge ferromagnetic resonator without surface metallization and the second structure is a trapped energy resonator (an energy trapping is due to a metallic region on a surface of an unrestricted ferromagnetic film). When such a combination is possible, one can introduce separate terms and analyze an intersection of these terms depending on a certain parameter (or a system of parameters). Such an analysis used for a two-atom molecule shows, in particular, that two terms of identical symmetry do not have an intersection [52]. Can this conclusion be applied for an initial characterization of symmetry properties of a straight-edge ferromagnetic resonator and a trapped energy resonator and, as a result, the properties of a bianisotropic particle? The situation is not so simple. We can see that every bianisotropic particle is not a mechanical junction of straight-edge and trapped energy resonators. This makes it necessary to analyze the symmetry properties of a particle on the whole without any initial decomposition.

In this consideration, another problem also arises. As we have shown, to vary the average density of the energy in bianisotropic media, a necessary correlation between components of the electric and magnetic fields should take place.

For this reason, the interaction Hamiltonian (a part of the total Hamiltonian correlated with the external fields) in the Liouville equation cannot be considered as a superposition of the ''electric'' and ''magnetic'' parts and should be introduced as a ''whole'' notion of the magnetoelectric interaction Hamiltonian.

V. CONCLUSION

It is evident that the electrodynamics of bianisotropic media is a very topical sphere of research. The fact that in a medium we may have an additional (with respect to the Maxwell equations) coupling between the electric and magnetic fields promises to produce very attractive fundamental problems. One has to be sure, however, that the results obtained and published in this sphere of research are really based on the main theoretical principles of macroscopic electrodynamics. So discussions about the main theoretical propositions of the macroscopic electrodynamics of bianisotropic media are very urgent. In this paper we tried to formulate some of these principles and to direct attention to bianisotropic composite materials that can meet the requirements of the laws of the macroscopic electrodynamics.

[1] I. V. Lindell, A. H. Sihvola, S. A. Tretyakov, and A. J. Viitanen, *Electromagnetic Waves in Chiral and Bi-Isotropic Media* (Artech House, Boston, 1994).

[2] A. Lakhtakia, *Beltrami Fields in Chiral Media* (World Scientific, Singapore, 1994).

[3] H. Cory, J. Electromagn. Waves Appl. **9**, 805 (1995).

[4] *Advances in Complex Electromagnetic Materials*, edited by A. Priou, A. Sihvola, S. Tretyakov, and A. Vinogradov (Kluwer Academic, Dordrecht, 1997).

[5] *Proceedings of Bianisotropics '97: International Conference and Workshop on Electromagnetics of Complex Media*, edited by W. S. Weiglhofer (University of Glasgow, Glasgow, 1997).

[6] F. N. H. Robinson, *Macroscopic Electromagnetism* (Pergamon, Oxford, 1973).

[7] J. D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1975).

[8] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, 2nd ed. (Pergamon, Oxford, 1984).

[9] V. L. Ginzburg, *The Propagation of Electromagnetic Waves in Plasmas* (Pergamon, Oxford, 1964).

[10] A. Gurevich and G. Melkov, *Magnetic Oscillations and Waves* (Nauka, Moscow, 1994).

[11] V. M. Agranovich and V. L. Ginzburg, *Crystal Optics with Spatial Dispersion and Excitons*, 2nd ed. (Springer-Verlag, New York, 1984).

[12] I. V. Lindell and A. H. Sihvola, Electron. Lett. **30**, 842 (1994).

[13] B. Jacoby and F. Olyslager, Electron. Lett. **31**, 779 (1995).

[14] B. Michel and A. Lakhtakia, Phys. Rev. E **51**, 5701 (1995).

[15] B. Shanker, Microwave Opt. Technol. Lett. **13**, 150 (1996).

[16] F. Olyslager, IEEE Trans. Antennas Propag. **42**, 1392 (1994).

[17] B. Jakoby, IEE Proc.-Microw. Antennas Propag. **143**, 527 (1996).

[18] X. Wu and D. L. Jaggard, IEEE Trans. Microwave Theory Tech. **45**, 640 (1997).

[19] V. V. Varadan, A. Lakhtakia, and V. K. Varadan, J. Appl. Phys. **63**, 280 (1988).

[20] A. Lakhtakia, V. K. Varadan, and V. V. Varadan, Appl. Opt. **29**, 3627 (1990).

[21] S. A. Tretyakov, F. Mariotte, C. R. Simovskii, T. G. Kharina, and J.-P. Heliot, IEEE Trans. Antennas Propag. **44**, 1006 (1996).

[22] E. O. Kamenetskii, Microwave Opt. Technol. Lett. **11**, 103 (1996).

[23] E. O. Kamenetskii, in *Advances in Complex Electromagnetic Materials* (Ref. [4]), pp. 227–238.

[24] E. O. Kamenetskii, Phys. Rev. E **57**, 3563 (1998).

[25] E. O. Kamenetskii, preceding paper, Phys. Rev. E **58**, 7959 (1998).

[26] L. Tsang and J. A. Kong, Radio Sci. **16**, 303 (1981).

[27] N. P. Zhuck, Phys. Rev. B **50**, 15 636 (1994).

[28] M. Barthelemy, H. Orland, and G. Zerah, Phys. Rev. E **52**, 1123 (1995).

[29] L. D. Landau and E. M. Lifshitz, *Statistical Physics*, 3rd ed. (Pergamon, Oxford, 1980).

[30] M. Born and K. Huang, *Dynamic Theory of Crystal Lattices* (Clarendon, Oxford, 1954).

[31] E. O. Kamenetskii, Phys. Rev. E **57**, 3556 (1998).

[32] R. M. Hornreich and S. Shtrikman, Phys. Rev. **171**, 1065 (1968).

[33] E. O. Kamenetskii, Phys. Rev. E **54**, 4359 (1996).

[34] A. Lakhtakia and W. S. Weiglhofer, Int. J. Infrared Millim. Waves **17**, 1867 (1996).

[35] N. P. Zhuck and A. S. Omar, IEEE Trans. Antennas Propag. **44**, 1142 (1996).

[36] E. O. Kamenetskii, in *Proceedings of Bianisotropics '97: International Conference and Workshop on Electromagnetics of Complex Media* (Ref. [5]), pp. 201–204.

[37] L. Onsager, Phys. Rev. **37**, 405 (1931); **38**, 2265 (1931).

[38] H. B. G. Casimir, Rev. Mod. Phys. **17**, 343 (1945).

[39] S. A. Tretyakov, A. A. Sochava, D. Ya. Khaliullin, and V. V. Yatsenko, Microwave Opt. Technol. Lett. **15**, 260 (1997).

[40] V. A. Dmitriev, Electron. Lett. **34**, 532 (1998).

[41] T. H. O'Dell, *The Electrodynamics of Magneto-Electric Media* (North-Holland, Amsterdam, 1970).

[42] J. A. Kong, *Electromagnetic Theory* (Wiley, New York, 1990).

[43] C. W. Nan, Phys. Rev. B **50**, 6082 (1994).

[44] G. N. Borzdov, in *Advances in Complex Electromagnetic Materials* (Ref. [4]), pp. 71–82.

[45] G. N. Borzdov, in *Advances in Complex Electromagnetic Materials* (Ref. [4]), pp. 83–84.

[46] I. V. Lindell, A. H. Sihvola, P. Puska, and L. H. Ruotanen, Microwave Opt. Technol. Lett. **8**, 268 (1995).

[47] L. H. Ruotanen and A. Hujanen, J. Electromagn. Waves Appl. **11**, 21 (1997).

[48] I. E. Dzyaloshinskii, Sov. Phys. JETP **10**, 628 (1960).

[49] A. Lakhtakia and W. S. Weiglhofer, in Proceedings of IEEE International Symposium on Electromagnetic Compat., Atlanta, 1995 (unpublished).

[50] A. Lakhtakia, Int. J. Infrared Millim. Waves **15**, 1363 (1994).

[51] E. O. Kamenetskii, Microwave Opt. Technol. Lett. **11**, 99 (1996).
[52] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics: Non-Relativistic Theory*, 3rd ed. (Pergamon, Oxford, 1977).
[53] L. I. Schiff, *Quantum Mechanics*, 3rd ed. (McGraw-Hill, New York, 1968).
[54] Y. R. Shen, *The Principles of Nonlinear Optics* (Wiley, New York, 1984).
[55] B. R. Fisher and P. Mittelstaedt, Phys. Lett. A **147**, 411 (1990).
[56] J. L. Perez-Diaz, V. M. Perez-Garcia, and I. Gonzalo, Phys. Lett. A **160**, 453 (1991).